Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True   **FALSE** The number of ways there are to flip 2 heads out of 5 total flips is $P(5, 2)$ because the order of the coin flips matters.

> **Solution:** We say order doesn't matter to mean that when picking the **location** of the H's, the order in which we pick them does not matter and hence we should use C. So order always has to do with the order in which you choose things, not in the ordering compared to everything else.

2. **TRUE**   False It is impossible to devise an algorithm to solve the stable marriage problem if men can marry other men (the "roommate problem").

> **Solution:** Zvezda gave an example of a system of preferences such that there is no stable pairing which means that an algorithm cannot give you a stable pairing.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) How many ways can I buy 200 bubble teas from TeaOne for an event if there are 10 different options to choose from?

> **Solution:** There are 200 indistiguishable balls which are the bubble teas, and the boxes are the options. So there are $\binom{200+10-1}{200}$ ways to do this.

  (b) (4 points) The most popular option is original milk tea. How many ways can I do this if I need at least 20 of that option and at least 10 of every other option?

> **Solution:** First I buy 20 of that option and 10 of all the other options. So I've bought 110 bobas and need to buy 90 more out of the 10 options. There are $\binom{90+10-1}{90}\binom{99}{90}$ ways to do this.

(c) (2 points) Suppose men and women have the preferences $m_1 : w_1 > w_3 > w_2, m_2 :$ $w_3 > w_1 > w_3, m_3 : w_3 > w_2 > w_1$ and $w_1 : m_1 > m_2 > m_3, w_2 : m_2 > m_3 > m_1, w_3 : m_3 > m_2 > m_1$. Is the matching $(m_1, w_1), (m_2, w_3), (m_3, w_2)$ stable?

> **Solution:** This is not stable because m_3 prefers w_3 to w_2 and w_3 prefers m_3 to m_2 which means that both with divorce their partners and get with each other.